Protection of Electrical Networks
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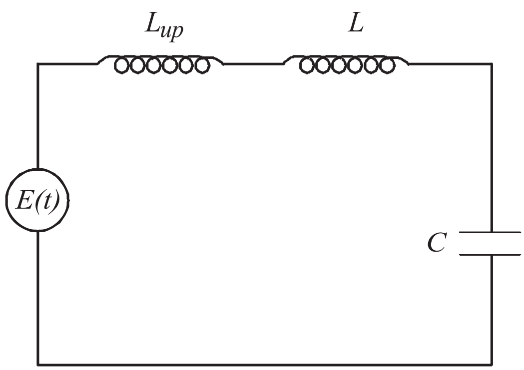
Appendix B

Calculation of Inrush Current During Capacitor Bank Energization

Fixed bank

The equivalent upstream network single-phase diagram during energization of the fixed bank is shown in Figure B-1.



$E(t)$: single-phase voltage

L_{up} : upstream network inductance

L : inductance of the connection linking the switching device to the capacitor bank

Figure B-1: *equivalent diagram during fixed bank energization*

We shall demonstrate that the frequency of the transient current occurring upon energization is very high (see section 10.6.1, example 1; $f_0 = 1{,}582 \text{ Hz}$).

This results in justification of neglect of the network resistance in relation to the inductance: $R_{up} << 2\,\pi\, f_0\, L_{up}$, since $f_0 >> 50 \text{ Hz}$.

Similarily, the resistance of the connection linking the switching device to the capacitor is negligible.

The network frequency (50 Hz) is negligible in relation to the transient current frequency. We might therefore consider that we have a voltage step throughout the duration of the transient current.

The value of the step, at worst, is the peak value of the sinusoidal voltage:

$$\hat{E} = \sqrt{2}\,\frac{U_n}{\sqrt{3}}$$

U_n : phase-to-phase voltage

The current $i(t)$ is determined by the following differential equation:

$$E(t) = \left(L_{up} + L\right)\frac{di}{dt} + \frac{1}{C}\int i$$

where: $\begin{cases} E(t) = 0 & \text{for} \quad t < 0 \\ E(t) = \hat{E} & \text{for} \quad t \geq 0 \end{cases}$

We shall solve this equation using Laplace transforms.

As a Laplace transform, the differential equation becomes:

$$\frac{\hat{E}}{s} = \left(L_{up} + L\right)\left[s\, I(s) - i(t=0)\right] + \frac{1}{Cs}\, I(s) - \frac{V_C(t=0)}{s}$$

The current is zero before energization and it is assumed that the voltage at the capacitor terminals is zero (worst case).

Hence: $i(t=0) = 0$ and $V_C(t=0) = 0$

thus giving us: $\frac{\hat{E}}{s} = \left(L_{up} + L\right) s\, I(s) + \frac{1}{Cs} I(s)$

hence: $$I(s) = \frac{\hat{E}}{s\left[\left(L_{up} + L\right) s + \frac{1}{Cs}\right]} = \frac{\hat{E}}{\left(L_{up} + L\right)\left[s^2 + \frac{1}{C\left(L_{up} + L\right)}\right]}$$

Let us take: $\omega = \frac{1}{\sqrt{C\left(L_{up} + L\right)}}$

$$I(s) = \frac{\hat{E}}{\left(L_{up} + L\right)\omega} \quad \frac{\omega}{s^2 + \omega^2}$$

Using the Laplace transform tables, we can deduce $i(t)$:

$$i(t) = \frac{\hat{E}}{\left(L_{up} + L\right)\omega} \sin \omega t$$

$$i(t) = \sqrt{\frac{2}{3}}\, U_n \sqrt{\frac{C}{L_{up} + L}} \sin \omega t$$

The maximum peak inrush current is thus:

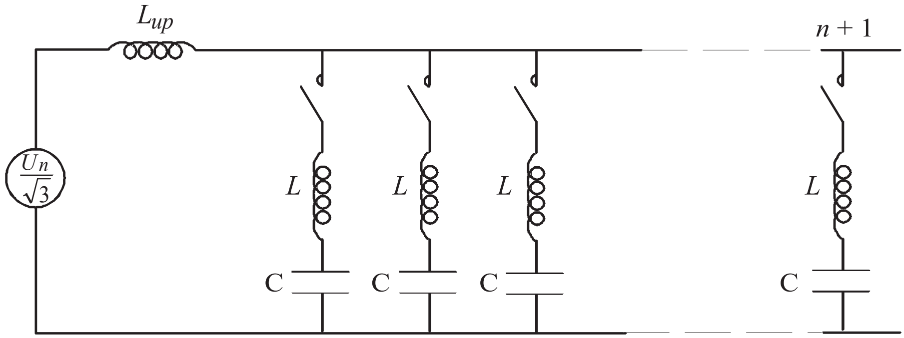
$$\hat{I}_{rush} = \sqrt{\frac{2}{3}}\, U_n \sqrt{\frac{C}{L_{up} + L}}$$

A and its frequency:

$$f_0 = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{C\left(L_{up} + L\right)}}$$

Switched steps bank

The equivalent single-phase diagram during switched steps bank energization is shown in Figure B-2.



L_{up} : upstream network inductance

L : inductance of the connection linking the switching device to the bank

Figure B-2: *equivalent diagram during switched steps bank energization*

The peak inrush current $\hat{I}_{rush}$ is maximum when n banks are in service and the $(n+1)^{th}$ one is energized. The banks in service off load into the bank that has just been energized.

The upstream inductance is very high in relation to inductance L (see section 10.6.1, example 1: $L_{up} = 385\ \mu H$ and example 2: $L = 2.5\ \mu H$). The current supplied by the upstream part (network) is therefore neglected.

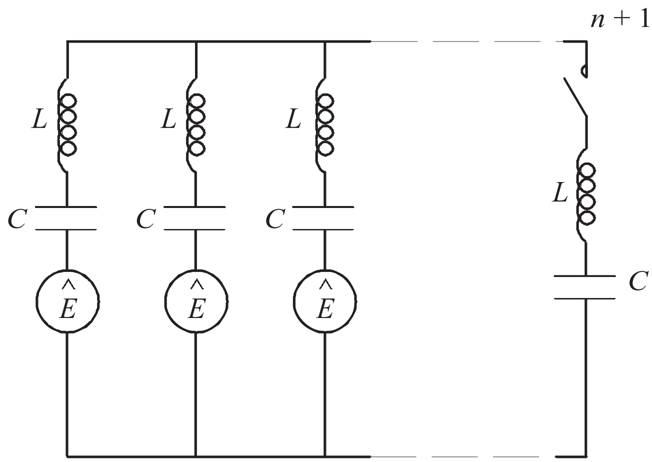
It is assumed that, at worst, upon energization the voltage at the terminals of each capacitor is $V_C\ (t=0) = \hat{E} = \sqrt{2}\ \dfrac{U_n}{\sqrt{3}}$.

The equivalent diagram is thus shown in Figure B-3.

The diagram comprises n parallel-connected branches with an impedance of $Z = j\ L\ \omega + \dfrac{1}{j\ C\ \omega}$.

The equivalent impedance is therefore:

$$Z_{eq} = \frac{Z}{n} = j\ \frac{L}{n}\ \omega + \frac{1}{j\ n\ C\ \omega}$$



$\hat{E}$: initial voltage condition at the capacitor terminals

Figure B-3

The diagram thus becomes that of Figure B-4.

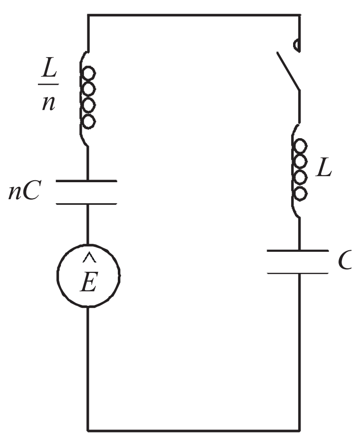


Figure B-4

We have two series-connected inductances: $\frac{L}{n} + L = L\left(\frac{n+1}{n}\right)$

We have two series-connected capacitances:

$$\frac{1}{C} + \frac{1}{n\,C} = \frac{1}{C}\left(\frac{n+1}{n}\right) = \frac{1}{C\left(\frac{n}{n+1}\right)}$$

The equivalent diagram is thus that in Figure B-5.

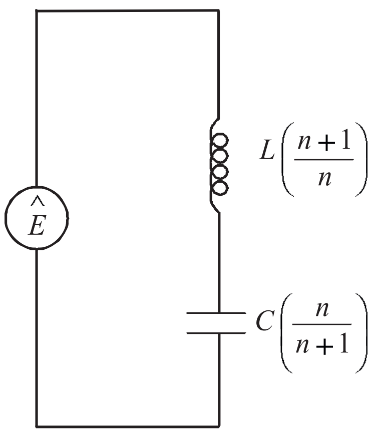


Figure B-5

The equivalent diagram in Figure B-5 is the same as that of a fixed bank.

If we re-use the formula for a fixed bank, we immediately obtain:

$$\hat{I}_{rush} = \sqrt{\frac{2}{3}} U_n \sqrt{\frac{C\left(\frac{n}{n+1}\right)}{L\left(\frac{n+1}{n}\right)}}$$

$$\hat{I}_{rush} = \sqrt{\frac{2}{3}} U_n \frac{n}{n+1} \sqrt{\frac{C}{L}}$$

$$f = \frac{1}{2\pi\sqrt{L\,C}}$$